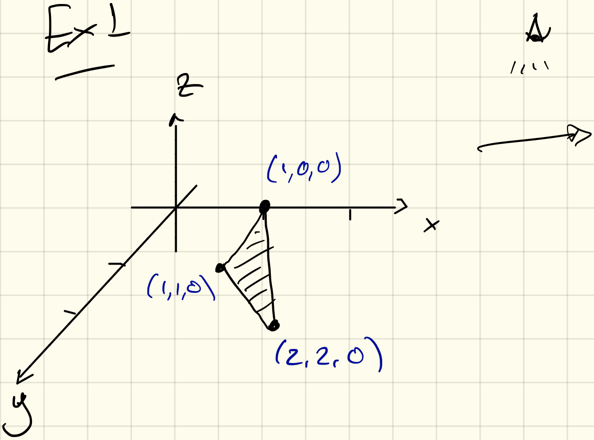
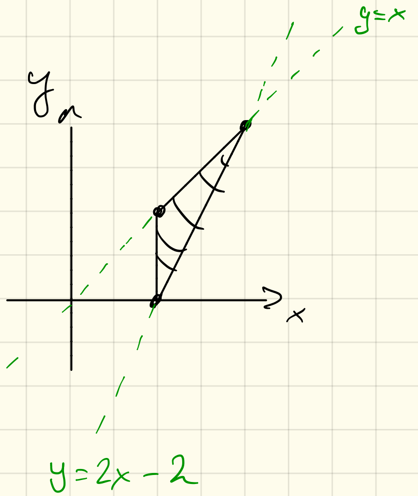
Série 9 :

Ex 1





$\sigma(x,y) = (x, y, 0)$,

$x \in [1,2]$, $y \in [2x-2, x]$

↳ comme dans le livre

Technique Ex4 série 1

$$\sigma(t_1,t_2)$$

$$= t_1(1,0,0) + t_2(1,1,0) + (1-t_1-t_2)(2,2,0)$$

$$= (t_1+t_2+2-2t_1-2t_2,\ t_2+2-2t_1-2t_2,\ 0)$$

$$= (2-t_1-t_2,\ 2-2t_1-t_2,\ 0)$$

$t_1 \in [0,1]$, $t_2 \in [0, 1-t_1]$.

$$\sigma_{t_1} = (-1,-2,0) \quad , \quad \sigma_{t_2} = (-1,-1,0)$$

$$\sigma_{t_1} \wedge \sigma_{t_2} = \begin{vmatrix} e_1 & e_2 & e_3 \\ -1 & -2 & 0 \\ -1 & -1 & 0 \end{vmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

$$\iint_{\Sigma_1} \operatorname{rot} F \, ds = \int_0^1 \int_0^{1-t_1} \langle (0,0,2(2-t_1-t_2)) ; (0,0,-1) \rangle \, dt_2 \, dt_1$$

$$= \int_0^1 \int_0^{1-t_1} 2t_1 + 2t_2 - 4 \, dt_2 \, dt_1$$

$$= \int_0^1 (2t_1-4)(1-t_1) + \left[t_2^2 \right]_{t_2=0}^{t_2=1-t_1} dt_1$$

$$= \int_0^1 2t_1 - 2t_1^2 - 4 + 4t_1 + (1-t_1)^2 \, dt_1$$

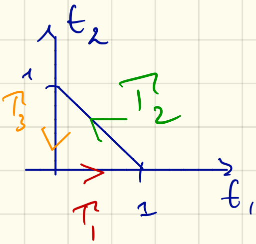
$$= \int_0^1 -2t_1^2 + 6t_1 - 4 + t_1^2 - 2t_1 + 1 \, dt_1$$

$$= \int_0^1 - t_1^2 + 4t_1 - 3 \, dt_1 = \left[-\frac{1}{3} t_1^3 + 2t_1^2 - 3t_1 \right]_{t_1=0}^{t_1=1}$$

$$= -\frac{1}{3} + 4 - 5 = -\frac{4}{3}$$

$\partial \Sigma_1$:

(méthode série 1 Ex4)



T_1 : $\gamma_1(t) = (t,0)$, $\sigma \circ \gamma_1(t) = \sigma(t,0) = (2-t, 2-2t, 0)$ $t \in [0,1]$

$$(\sigma \circ \gamma_1)'(t) = (-1,-2,0)$$

$$\int_{\sigma(T_1)} F \cdot dl = \int_0^1 \langle (0, (2-t)^2, 0) ; (-1,-2,0) \rangle \, dt$$

$$= \int_0^1 -2(2-t)^2 \, dt = \int_0^1 -2t^2 + 8t - 8 \, dt$$

$$= \left[-\frac{2}{3}t^3 + 4t^2 - 8t \right]_{t=0}^{t=1} = -\frac{2}{3} + 4 - 8 = -\frac{14}{3}.$$

T_2 : $\gamma_2(t) = (1-t, t)$, $\sigma \circ \gamma_2(t) = (2-(1-t) - t, 2-2(1-t) - t, 0)$

$$= (1, t, 0) \quad t \in [0,1]$$

$$(\sigma \circ \gamma_2)'(t) = (0,1,0)$$

$$\int_{\sigma(T_2')} F \cdot dl = \int_0^1 \langle (0,1,0), (0,1,0) \rangle dt = \int_0^1 1 \, dt = 1$$

T_3' : $\gamma_3(t) = (0, 1-t)$, $\sigma \circ \gamma_3(t) = (2-(1-t), 2-(1-t), 0)$

$$= (t+1, t+1, 0) \qquad t \in [0,1]$$

$$\sigma \circ \gamma_3'(t) = (1,1,0)$$

$$\int_{\sigma(T_3')} F \cdot dl = \int_0^1 \langle (0, (t+1)^2, 0) ; (1,1,0) \rangle dt$$

$$= \int_0^1 t^2 + 2t + 1 \, dt = \left[\frac{1}{3} t^3 + t^2 + t \right]_{t=0}^{t=1}$$

$$= \frac{7}{3}.$$

$\boxed{\text{OU}}$ $\gamma_3(t) = (0, t)$ $t : 1 \to 0$ $-$

$$\sigma \circ \gamma_3(t) = (2-t, 2-t, 0), \quad (\sigma \circ \gamma_3)'(t) = (-1,-1,0)$$

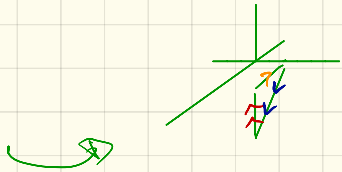
$$\int_{\sigma(T_3')} F \cdot dl = -\int_0^1 \langle (0, (2-t)^2, 0) ; (-1,-1,0) \rangle dt$$

$$= \int_0^1 t^2 - 4t + 4 \, dt = \left[\frac{1}{3} t^3 - 2t^2 + 4t \right]_0^1 = \frac{1}{3} - 2 + 4$$

$$= \frac{7}{3}$$

$\partial \Sigma_1$

(méthode par dessin)



$\sigma(x,y) = (x,y,0)$, $x \in [1,2]$,

$y \in [2x-2, x]$.

T_1 : $\gamma_1(t) = (t, 2t-2)$, $\sigma\circ\gamma_1(t) = (t, 2t-2, 0)$ $t \in [1,2]$

$$(\sigma \circ \gamma_1)'(t) = (1,2,0)$$

$$\int_{\sigma(T_1)} F \cdot dl = \int_1^2 \langle (0, t^2, 0) ; (1,2,0) \rangle dt = \int_1^2 2t^2 dt$$

$$= \left[\frac{2}{3} t^3\right]_1^2 = \frac{16}{3} - \frac{2}{3} = \frac{14}{3}$$

T_2 : $\gamma_2(t) = (2-t, 2-t)$, $\sigma\circ\gamma_2(t) = (2-t, 2-t, 0)$ $t \in [0,1]$.

$$(\sigma \circ \gamma_2)'(t) = (-1,-1,0)$$

$$\int_{\sigma(T_2)} F \cdot dl = \int_0^1 \langle (0, (2-t)^2, 0) ; (-1,-1,0) \rangle dt$$

$$=\int_0^1 -t^2+4t-4\,dt = \left[-\frac{1}{3}t^3+2t-4t\right]_0^1$$

$$=-\frac{1}{3}+2-4=-\frac{7}{3}$$

$\boxed{\text{(oro}}$ $\gamma_2(t)=(t,t)$, $t: 2\to 1$ $\ominus$

$$\sigma\circ\gamma_2(t)=(t,t,0).\quad (\sigma\circ\gamma_2)'(t)=(1,1,0)$$

$$\int_{\sigma(T_2)} F\cdot dl = -\int_1^2 \langle (0,t^2,0);(1,1,0)\rangle\,dt$$

$$=-\left[\frac{1}{3}t^3\right]_{t=1}^{t=2} = -\frac{8}{3}+\frac{1}{3}=-\frac{7}{3}$$

T_3: $\gamma_3(t)=(1,1-t)$, $\sigma\circ\gamma_3(t)=(1,1-t,0)$ $t\in[0,1]$

$$(\sigma\circ\gamma_3)'(t)=(0,-1,0)$$

$$\int_{\sigma(T_3)} F\cdot dl = \int_0^1 \langle (0,1,0);(0,-1,0)\rangle\,dt = -1$$

$\boxed{\text{ou}}$ $\gamma_3(t)=(1,t)$ $t: 1\to 0$. $\ominus$

$$\sigma\circ\gamma_3(t)=(1,t,0),\quad (\sigma\circ\gamma_3)'(t)=(0,1,0)$$

$$\int_{\sigma(C_3)} F \cdot d\ell = -\int_0^1 \langle (0,1,0) ; (0,1,0) \rangle \, dt = -1$$